# Electroproduction of Neutral Pions from Deuterium*†

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Making use of the impulse approximation and the one-nucleon dispersion theoretical amplitudes, a theoretical expression for the differential cross section of the process $e+d \rightarrow e+d+\pi^0$ has been calculated. An experiment which would be useful in the investigation of the nucleon form factors is proposed. At energies near the (3,3) resonance, the cross sections reach values up to $10^{-35}$ cm$^2$/MeV sr$^2$, depending on the electronic four-momentum transfer $\lambda^2$.

## I. INTRODUCTION

RECENT experiments in high-energy electron scattering,[1] coupled with the dispersion relation approach to the theory of $\pi$-meson production[2,3] and with the use of the impulse approximation for treating systems of two or more nucleons,[4] have now made it possible to conduct a theoretical study of electroproduction of pions from deuterium. This theory, in conjunction with a suitably designed experiment, is capable of yielding new information on the structure of nucleons.

Even the relatively simple case treated here of coherent electroproduction of pions from deuterium, in which the deuteron remains bound in the final state, is interesting by reason of the relatively simple way in which the nucleon form factors appear. Moreover, it can provide information on the deuteron form factors and, hence, on the internucleon potential. Especially in this latter respect, it is helpful in the following to make use of the analogous situation of photoproduction from deuterium.[5] Following the example of Ref. 5, we make no attempt to analyze the possibility of breaking up the deuteron into two nucleons, with the production of charged mesons as well as neutral ones, even though experimental information about the sum of the four possible one-pion processes is available.[6] Such an analysis, of course, provides a natural extension of this work, but offers additional difficulties in making phase-space calculations and in describing the final state.

Although the principal concern of the calculation is the evaluation of the matrix element, a word is in order here about the experimental arrangement we are proposing; this will explain the phase-space calculations and the kinematics which are involved in the final laboratory cross sections that are our numerical results. We postulate a situation in which the incident electron energy is known, and the final electron and deuteron in coincidence are measured both in energy and in solid angle of scattering. This information would seem to overdetermine the kinematics by one parameter, say, the final deuteron energy, but one finds it necessary to distinguish between the two solutions of the quadratic momentum-energy conservation equations which are well known to exist for this type of inelastic process. Moreover, the overdetermination serves as a unique signature of single pion electroproduction and, therefore, helps to separate that process from the background which has caused many difficulties in other experiments. Figure 1 is a diagram showing the kinematical parameters determining the cross section.

## II. ANALYSIS

The phase-space calculations for electroproduction of pions from nucleons have been performed by Dalitz and Yennie,[7] who have also discussed the kinematical procedures for treating systems with two initial and three final particles. With minor variations due to the deuteron as target particle and the different experimental arrangement, their arguments hold also for the present discussion. Hence, the laboratory differential cross section for electroproduction from deuterium may be written[8]

$$\frac{d\sigma}{dR_{20}d\Omega_R d\Omega_P}=\frac{\Phi}{(2\pi)^5}\frac{m^2M^2}{2M^d}\frac{p_{10}{}^d p_{20}{}^d}{p_{10}p_{20}}\frac{R_2}{R_1}\times\frac{(P_2{}^d)^2}{(K_0+M^d)P_2{}^d-\mathbf{K}\cdot\hat{P}_2{}^d P_{20}{}^d}. \quad (1)$$

The masses $m$, $\mu$, $M$, and $M^d$ are those of the electron, meson, nucleon, and deuteron, respectively. In general, lower case letters represent energies or momenta in either an unspecified coordinate system or in the pion-deuteron center of mass (c.m.) system, while capitals refer to the laboratory system. Subscripts 1 and 2 designate initial and final quantities for the electron,

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[8] We use the rationalized natural units: $\hbar=1=c$, $e^2/4\pi\approx 1/137$, $g^2/4\pi\approx 14$, $f=(\mu/2M)g$.

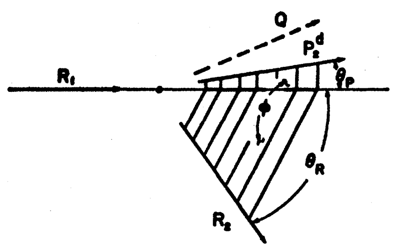


FIG. 1. Kinematical quantities in the laboratory for electroproduction from deuterium. The meson momentum **Q** is not measured, but is included in the figure for completeness. The dot represents the initial deuteron at rest. Notice that the three final momenta are not necessarily coplanar.

deuteron, and interacting nucleon, which have four-momenta $r_\mu$, $p_\mu{}^d$, and $p_\mu$, respectively. The pion is denoted by $q_\mu$, and the electronic four-momentum transfer is $k_\mu = r_{1\mu} - r_{2\mu}$, with $k_\mu k_\mu = \lambda^2$. Finally, the quantity $\Phi$ is the average over initial states and sum over final states of the square modulus of the matrix element $T$. We, therefore, write

$$\Phi = \tfrac{1}{6} \sum |T|^2, \qquad (2)$$

where the sum extends over all spins.

The ratio of deuteron to nucleon energies appears as a factor in Eq. (1) because in using the impulse approximation we will be dealing with matrix elements normalized by nucleon spinors. Therefore, a volume of normalization consistent with nucleon energies must be used rather than one which would be appropriate for deuterons. We shall see later that the denominator of this ratio must be averaged over the internal momentum of the deuteron along with several other quantities.

It is convenient to follow the methods of Dalitz and Yennie[7] in separating the matrix element into the four product of a deuteron four-current matrix element $j_\mu{}^d$ with the Møller potential $A_\mu$ describing the interaction of the electron and the virtual photon of momentum $k_\mu$. In this form it is a straightforward task to apply ordinary Dirac theory with positive energy projection operators to carry out the electron spin sum. On the other hand, the sum over deuteron spins can be performed if we use the fact that the deuteron current matrix element is to be taken, in the impulse approximation, as the sum of single nucleon amplitudes. Since the single nucleon current operators are expressed in terms of the nonrelativistic Pauli matrices, it is only necessary to insert the triplet spin-one projection operator $T_{ab}$ appropriately in order to invoke closure and express the sum as a trace (Tr) over these matrices. The result of these manipulations is that we can write

$$\Phi = \frac{\pi\alpha}{3m^2\lambda^2} \mathrm{Tr}\left[ \bar{j}_\mu{}^d T_{ab} j_\mu{}^d T_{ab} + \frac{\bar{r}_\mu j_\mu{}^d T_{ab} (\bar{r}_\mu j_\mu{}^d)^\dagger T_{ab}}{\lambda^2} \right], \qquad (3)$$

where we use the definitions

$$\bar{j}_\mu{}^d j_\mu{}^d = \mathbf{j}^{d\dagger} \cdot \mathbf{j}^d - |j_0{}^d|^2 \qquad (4)$$

and

$$\bar{r}_\mu = r_{1\mu} + r_{2\mu}. \qquad (5)$$

The symbol $j_\mu{}^d$ now stands for the deuteron current operator as expressed in terms of the nucleon current operators.

At this point, one further simplification is possible. As discussed by Fubini, Nambu, and Wataghin,[3] the one-nucleon amplitude may be decomposed into three isotopic spin components corresponding roughly to the three charge states in which the pion can be produced. Because we are only interested in the production of neutral pions, we find that the minus component as defined in Ref. 3 does not contribute; moreover, the deuteron is an isotopic singlet combination of the two isospin $\frac{1}{2}$ nucleons, and the zero component vanishes under the application of singlet isospinors in both initial and final states. The latter fact represents an important advantage over electroproduction from nucleons in that we now have only to deal with the plus amplitude. Therefore, we can use for $j_\mu{}^d$ in Eq. (13) only that part of the deuteron current operator deduced from the plus components of the one-nucleon amplitudes.

In order to be able to write down an expression for the deuteron current operator, we must now apply the critical assumption of the impulse approximation.[4] In this approximation, each nucleon contributes independently to the total amplitude as if it alone were interacting with the incident virtual photon to produce the pion. The only effect of one nucleon on the contribution of the other is due to the sharing of absorbed momentum. This effect will be shown to lead simply to the appearance of the deuteron form factors in the amplitude. Hence, the final amplitude can be obtained by summing the amplitudes due to each of the nucleons in the deuteron. We ignore the possibility of multiple scattering, which is expected to alter the over-all magnitude of the cross sections without having a large effect on their shapes.[5]

That these assumptions are reasonably well justified may be argued from the following observations. First, the binding energy of the deuteron is small compared with the energies of interaction and should not lead to significant correlations between the nucleons during the interaction. Second, the typical range for electroproduction seems to be less than the average separation of the nucleons, so that the production is in some sense localized. Finally, the nucleons usually have low momentum relative to the deuteron c.m. system and do not move appreciably during the characteristic time for the interaction.

The single nucleon amplitudes which are to be added to give the deuteron amplitude depend, of course, on the momenta of the individual nucleons before and after the collision. Since these are experimentally indeterminable quantities, it is necessary to average over the internal momenta of the deuteron before and after the collision—$\mathbf{s}_1$ and $\mathbf{s}_2$, respectively—using the deuteron momentum wave function $\phi_d$ as a weight. For instance, for nucleon $a$ with amplitude $T_a$ we would write

$$(T_a)_{av}=\int d^3s_1\,\phi_d^\dagger(\mathbf{s}_2)T_a(\mathbf{s}_1,\mathbf{s}_2)\phi_d(\mathbf{s}_1)\,, \qquad (6)$$

where the initial and final momenta are related by the fact that the spectator nucleon must conserve its momentum, so that we have

$$\mathbf{s}_2=\mathbf{s}_1+\mathbf{p}_t/2\,,\quad \mathbf{p}_t=\mathbf{p}_2{}^d-\mathbf{p}_1{}^d. \qquad (7)$$

Expression (6) is difficult to evaluate in that it would be necessary to carry out the integration numerically for each set of kinematic inputs. We observe, however, that the amplitudes are rather insensitive to variations in $\mathbf{s}_1$, whereas the deuteron wave functions are sharply peaked about relative momentum of zero. Therefore, we make the simplifying assumption that we can replace the matrix element by its average between $\mathbf{s}_1=0$ and $\mathbf{s}_2=0$. It is then possible to convert Eq. (6) to relatively simple integrals in configuration space over the spatial wave functions, $\psi_d$.

Furthermore, since the Møller potential part of the amplitude depends only on electron coordinates, it does not enter into this averaging process as performed in the pion-deuteron c.m. system. Hence, the foregoing arguments can be applied equally well to just the current part of the problem, so that we can write

$$j_\mu{}^d=\int d^3x\,\psi_d^\dagger(\mathbf{x})e^{i\mathbf{p}_t\cdot\mathbf{x}/2}(\bar{j}_{a\mu}+\bar{j}_{b\mu})\psi_d(\mathbf{x})\,, \qquad (8)$$

where

$$\bar{j}_{a\mu}=\tfrac{1}{2}[j_{a\mu}(\mathbf{s}_1=0)+j_{a\mu}(\mathbf{s}_1=-\mathbf{p}_t/2)]. \qquad (9)$$

## III. RESULTS

We are now in a position to complete the evaluation of the differential cross section for electroproduction from deuterium. Although the calculations are in practice algebraically complicated, they are in principle straightforward. In this section we, therefore, limit ourselves to the explicit description of the input data and to the presentation of the numerical results.

The one-nucleon matrix element has been calculated by several authors.[3,9,10] We use a relatively simple result derived from dispersion relations in a static approximation by Fubini, Nambu, and Wataghin.[11] It is relatively easy to extract from their plus amplitude the nucleon current in the pion-nucleon c.m. system. This must then be converted to the pion-deuteron c.m. system before it can be summed and averaged. Whenever possible, the transformation is expressed as an expansion in powers of the Galilean relative velocity, and only the lowest order corrections are retained. This allows the average to be carried out in a trivial manner, except in the case of the argument of the phase shift $\delta_{33}$ and of the product $p_{10}p_{20}$, which are simply averaged between $\mathbf{s}_1=0$ and $\mathbf{s}_1=-\mathbf{p}_t/2$.

For the $j=\frac{3}{2}$, $I=\frac{3}{2}$ pion-nucleon scattering phase shift $\delta_{33}$ we use an empirical relation derived from one given by Ball.[12] In terms of $\mathbf{q}'$, the averaged pion momentum in the pion-nucleon c.m. system, and $W'$, the associated total c.m. energy, the phase shift is given by

$$\sin^2\delta_{33}=1+\Gamma(W'^2-W_R{}^2)^2(W'^2-M^2)^2(q')^{-6}. \qquad (10)$$

If units of inverse fermis are used,[13] then the constants are $W_R{}^2=38.5$ F$^{-2}$ and $\Gamma=6.97\times10^{-4}$ F$^2$.

The effective vector magnetic moment $\mu^v$ depends on the structure of the nucleons through the charge and magnetic moment form factors as functions of $\lambda^2$. If it is written as

$$\mu^v=(e/2m)F^v\,, \qquad (11)$$

then $F^v$ is related to the vector form factors by

$$F^v=F_1{}^v+3.70F_2{}^v. \qquad (12)$$

These latter functions have been fitted empirically by de Vries, Hofstadter, and Herman to the form[14]

$$F_{1,2}{}^v(\lambda^2)=1-v_{1,2}+v_{1,2}/(1+0.113\lambda^2)\,, \qquad (13)$$

where $\lambda^2$ must be given in F$^{-2}$ and $v_1=0.92$, $v_2=1.10$.

In evaluating the integrals in Eq. (8), in which the currents remain interior to the integral only because they contain operators acting on the deuteron spinors, we utilize the definitions of Hadjioannou for his functions $F_{uu}\cdots G_{ww}$.[5] The deuteron wave function used contains a 6.8% $D$ state portion due to tensor forces. The correspondence of these integrals with the deuteron form factors is also as given by Hadjioannou. The numerical values of the latter were calculated using an empirical fit to the curves of Fig. 6 of McIntyre and Dhar.[15] The final evaluation of the traces is then accomplished by repeated use of identities among the Pauli matrices.[16]

The final expression for the cross section is much too lengthy to be reproduced here. The calculations were programmed for a RPC-4000 computer using the PINT (Purdue Interpretive) system for a limited set of kinematical inputs. The three angles shown in Fig. 1 were all kept constant, and the dependencies on $\lambda^2$ and

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[10] Phillipe Dennery, Phys. Rev. **124**, 2000 (1961).

[11] See Eq. (15) of Ref. 3, which contains a typographical error. On the right-hand side, $f^2$ should read $f^2/4\pi$.

[12] James Stutsman Ball, Phys. Rev. **124**, 2014 (1961).

[13] One fermi = 1 F = $10^{-13}$ cm.

[14] C. de Vries, R. Hofstadter, and Robert Herman, Phys. Rev. Letters **8**, 381 (1962).

[15] John A. McIntyre and Sobhana Dhar, Phys. Rev. **106**, 1074 (1957).

[16] This procedure also follows that of Hadjioannou. However, there exist some errors in the smaller terms of his expression. These errors have been acknowledged by the author, who is now in agreement with our results.

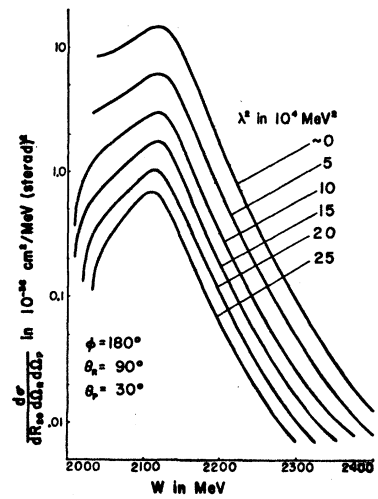


FIG. 2. Laboratory differential cross sections for electroproduction from deuterium. $W$ is the total energy in the c.m. system of the pion and deuteron and $\lambda^2$ is the four-momentum transfer from the electron. See Fig. 1 for the definition of the angles.

$W$, the total pion-deuteron c.m. energy, were generated by varying the final and initial electron energies. The values used are consistent with the physical limitations of the linear accelerator at Stanford.[17] The results are shown in Fig. 2 in terms of $\lambda^2$ and $W$.

## IV. CONCLUSIONS

The calculated values of the cross section, above $10^{-36}$ $cm^2/MeV\ sr^2$ for sufficiently low $\lambda$, $W$, and $p_t$, indicate that the proposed experiment is probably feasible, although it is clear that some care would have to be exercised in choosing the solid angles of acceptance and the size of the energy bins in order to assure a reasonable counting rate.

Even though one might expect fairly good agreement of our calculations with the results of such an experiment, say of order 20 or 30%, it is more interesting to consider what sort of analysis might be made by comparing the experimental results with the theory. There are two such analyses which seem most promising with respect to gaining new physical information.

The first of these is to follow the analysis of Hadjioannou.[5] One assumes that he knows the nucleon form factors sufficiently well and extracts an effective deuteron form factor as a function of $p_t$. This function can then be compared with ones calculated from various deuteron models and conclusions can be reached concerning the internucleon potential.

On the other hand, it is perhaps more useful to assume these deuteron form factors known, say as given by Hadjioannou's treatment, and instead try to obtain information on the nucleon form factors. From an examination of the expression given for the one-nucleon amplitudes[11] and the definitions (11) and (12), it is evident that the cross section is directly proportional to $(F^v)^2$. Therefore, one would need only divide the experimental cross sections by the proportionality factor—a function of the kinematics, phase shifts, and deuteron form factors—to obtain $(F^v)^2$ as a function of $\lambda^2$. Although this would give only the sum of the vector form factors, and moreover would leave $F^v$ ambiguous by a sign, it should provide one more combination of the form factors to add to those obtained from other electroproduction and electron-scattering experiments. Once a sufficient number of combinations have been collected, it will be possible to assign unambiguous values to all of the form factors, and to determine from them the nucleon charge distributions.

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[17] Edwin Erickson (private communication).